Thermodynamics inside fireballs and inside neutron stars

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The Modern Physics of Compact Stars

Dedicated to Victor A. Ambartsumyan 1908 - 2008

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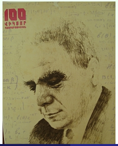
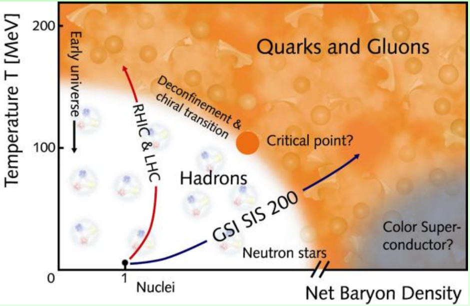
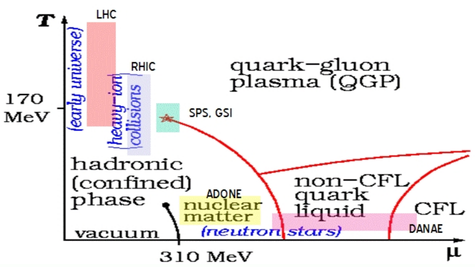


Are dense hadronic matters alike?





Starting points

Die Mesonenausbeute beim Beschuß von leichten Kernen mit α-Teilchen

Von Heinz Koppe

Max-Planck-Institut für Physik, Göttingen

(Z. Naturforschg. 3a, 251—252 [1948]; eingeg. am 21. Juni 1948)

Mittels des neuen Berkeley-Betatrons ist es möglich gewesen, durch Beschuß von leichten Kernen (insbesondere C) mit α-Teilchen von etwa 380 MeV Mesonen zu erzeugen. Im folgenden soll eine einfache Methode angegeben werden, nach der sich die dabei zu erwartende Ausbeute abschätzen läßt.

Beim Stoß eines Kernes mit der Massenzahl M_1 und der kinetischen Energie E auf einen ruhenden Kern mit der Masse M_2 entsteht zunächst ein Zwischenkern mit der Masse $M = M_1 + M_2$, dem die Anregungsenergie pro Nucleon

$$U = \frac{M_2}{M^2} E \qquad (1)$$

zur Verfügung steht. Nach einer bekannten Beziehung[1] hat der Zwischenkern dann die Temperatur

$$T_0 = 3{,}8 \sqrt{U}. \qquad (2)$$

Dabei wird unter T das Produkt aus k und der absoluten Temperatur verstanden. Gl. (2) liefert T in MeV, wenn man U in MeV einsetzt.

Application of statistical physics to elementary particles is usually referred to Enrico Fermi (1950)

although it was Heinz Koppe who proposed (1948) this idea to production processes

Die Ausbeute an Mesonen ist dann gegeben durch

$$\eta = \int_0^\infty \nu(T)\, dt = \frac{O\,\mu}{\pi^2 \hbar^3} \int_0^\infty T^2 e^{-\mu c^2 \sqrt{1/T_0^2 + 2Bt}}\, dt\,.$$

Unter dem Integral kann man T^2 als langsam veränderlich durch T_0^2 ersetzen und außerdem die Wurzel nach t entwickeln. Es ergibt sich

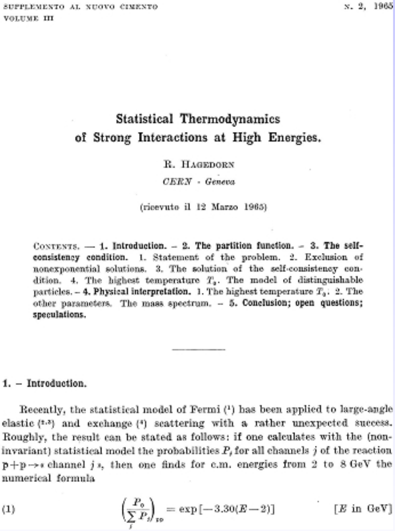
$$\eta = 0{,}031\, T_0\, M\, e^{-\mu c^2 / T_0}\,. \qquad (7)$$

Mit den oben angegebenen Werten liefert das Stoßausbeuten $\eta = 1{,}7 \cdot 10^{-4}$.

Limiting temperature

Rolf Hagedorn was the first who systematically analyzed high energy phenomena using all tools of statistical physics. He introduced the concept of the limiting temperature $\sim 140 MeV$ based on the statistical bootstrap model.

That was the origin of multiphase structure of hadronic matter.


SUPPLEMENTO AL NUOVO CIMENTO VOLUME III — N. 2, 1965

Statistical Thermodynamics of Strong Interactions at High Energies.

R. HAGEDORN

CERN - Geneva

(ricevuto il 12 Marzo 1965)

CONTENTS. — **1. Introduction.** – **2. The partition function.** – **3. The self-consistency condition.** 1. Statement of the problem. 2. Exclusion of nonexponential solutions. 3. The solution of the self-consistency condition. 4. The highest temperature T_0. The model of distinguishable particles. – **4. Physical interpretation.** 1. The highest temperature T_0. 2. The other parameters. The mass spectrum. – **5. Conclusion; open questions; speculations.**

1. – Introduction.

Recently, the statistical model of Fermi (1) has been applied to large-angle elastic (2,3) and exchange (4) scattering with a rather unexpected success. Roughly, the result can be stated as follows: if one calculates with the (non-invariant) statistical model the probabilities P_j for all channels j of the reaction $p+p \to s$ channel j s, then one finds for c.m. energies from 2 to 8 GeV the numerical formula

$$\left(\frac{P_0}{\sum_j P_j}\right)_{pp} = \exp[-3.30(E-2)] \qquad [E \text{ in GeV}] \quad (1)$$

Theoretical description of particle production

$$\mathcal{P}_n(i \to f) =$$

$$\int d^4 p'_1 \dots d^4 p'_n \delta(p'_1 + \cdots + p'_n - P_i) \prod_{j=1}^{n} \delta(p'^2_j - m_j^2) |\langle p'_1, \dots p'_n | \mathcal{S} | i \rangle|^2$$

The dynamical part

$$\langle p'_1, \dots p'_n | \mathcal{S} | i \rangle$$

The kinematical part

$$\delta(p'_1 + \cdots + p'_n - P_i) \prod_{j=1}^{n} \delta(p'^2_j - m_j^2)$$

Place for statistical physics

More particles (degrees of freedom) in the process: kinematics tends to dominate the behavior of the system

- measurable quantities are much less detailed than $\langle p'_1, \dots p'_n | \mathcal{S} | i \rangle$
- with the integration over a large region of the phase space the dynamical details are averaged and only a few parameters remains
- restricted knowledge of $\langle p'_1, \dots p'_n | \mathcal{S} | i \rangle$ is not needed

$$P_n = \bar{S}_n \, \mathcal{R}_n$$

$$\mathcal{R}_n = \int d^4 p'_1 \dots d^4 p'_n \delta(p'_1 + \dots + p'_n - P_i) \prod_{j=1}^{n} \delta({p'}_j^2 - m_j^2)$$

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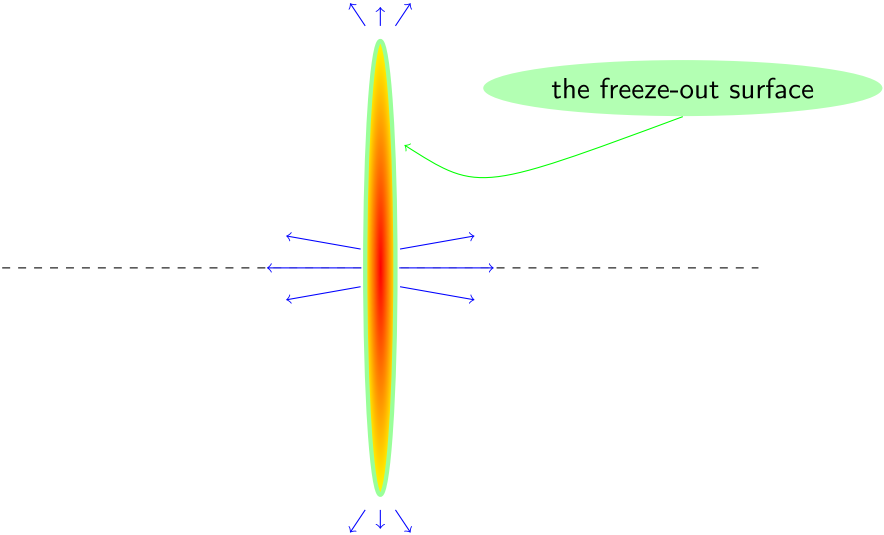
Arguments work if the thermodynamic equilibrium is reached

HIC collision – graphics



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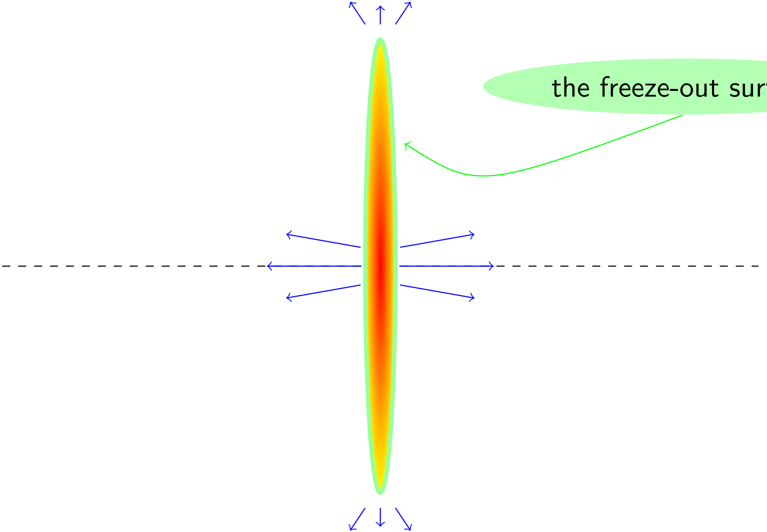
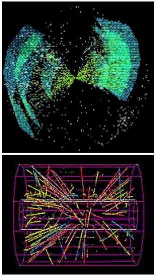




The aim of statistical models

To derive the equilibrium properties of a macroscopic system from the measured yields of the constituent particles

but

Not to describe how a system approaches equilibrium.

The chemical freeze-out

The stage where hadrons have been created and the net numbers of stable particles of each type no longer change in further evolution of the system.

Statistical model calculations – in principle

$$\epsilon = \frac{1}{2\pi^2} \sum_{j=1} (2S_j + 1) \int_0^\infty \frac{dp\, p^2 E_j}{\exp\left\{\frac{E_j - \mu_j}{T}\right\} + \lambda_j} \ ,$$

$$n_b = \frac{1}{2\pi^2} \sum_{j=1} (2S_j + 1) b_j \int_0^\infty \frac{dp\, p^2}{\exp\left\{\frac{E_j - \mu_j}{T}\right\} + \lambda_j} \ ,$$

$$n_s = \frac{1}{2\pi^2} \sum_{j=1} (2S_j + 1) s_j \int_0^\infty \frac{dp\, p^2}{\exp\left\{\frac{E_j - \mu_j}{T}\right\} + \lambda_j} \ ,$$

where

$$\mu_j = b_j \mu_b + s_j \mu_s$$

Almost reality

The time-space evolution of the system after collision is given by kinetic equations

long distance forces

$$(p_\mu \partial^\mu + F_\mu^{(j)} \partial^\mu (p)) f_j(x,p) = C_j(x,p)$$

interaction with other particles

phase space distribution function

Fewer degrees of freedom: small number of particles or low temperature: dynamics more and more important

Fireball parameters

The mean free path

$$\lambda_j = \frac{1}{\sum_k \sigma_{jk}\rho_k}$$

Between scattering time

$$\tau^{(j)}_{scatt} = \frac{\lambda_j}{\langle v_j \rangle} \sim \frac{1}{\sum_k \langle v_{jk}\sigma_{jk}\rangle}$$

Escape time

$$\tau^{(j)}_{esc} = \frac{R}{\langle v_j \rangle} \sim \frac{R}{c}$$

Expansion time

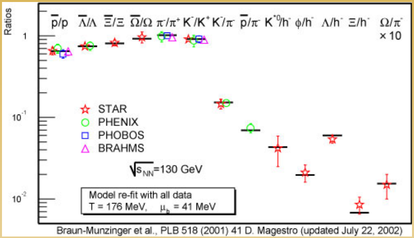
$$\tau_{exp} = \frac{1}{\partial_\mu u^\mu(x)}$$

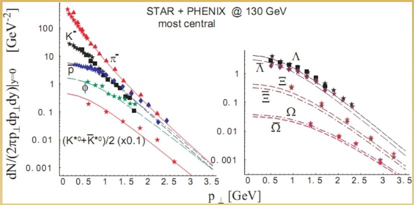
Freeze-out condition

$$\tau^{(j)}_{scatt} \gtrsim \min(\tau^{(j)}_{esc}, \tau_{exp})$$

gives

freeze-out hypersurface $\Sigma^{(j)}_f(x)$





although there are discussions is this "the real" thermodynamical temperature

$$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial U}$$

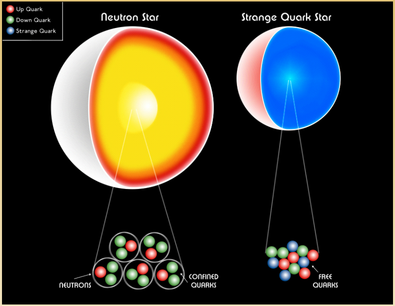
or "a fake" temperature due to the phase space dominance effect.

Puzzle

Multiproduction at high energy e^+e^- and pp is also well described within statistical model.

We cannot solve pre-equilibrium HIC dynamics, we have no good description of hadronization processes. . . Nevertheless, the thermal statistical models work quite well.

Differences



Principle difference between statistical model in HIC and in statistical model in the interior of neutron stars:

- High density in HIC does not lead to gravitational effects.
- High density in the interior of neutron star and the large mass force us to take into account gravitational effects.

Hydrostatic equilibrium

The neutron star matter is in its lowest energy state $\Rrightarrow$ thermodynamic equilibrium with respect to all reaction channels.

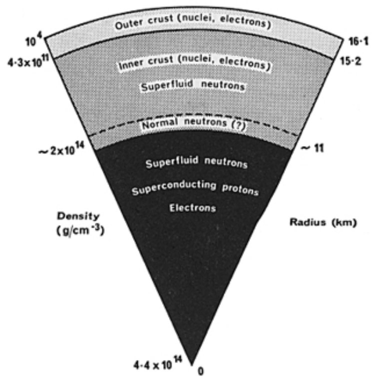
Hydrostatic equilibrium is decoupled from thermal evolution

Hydrostatic equilibrium

$$\begin{aligned} \frac{dm}{dr} &= 4\pi\rho r^2 \\ \frac{dP}{dr} &= -\frac{Gm\rho}{r^2}\,[\ldots\ \textit{General Relativity Corrections}\ \ldots] \\ P &= P(\rho,\ldots) \qquad (EoS) \end{aligned}$$

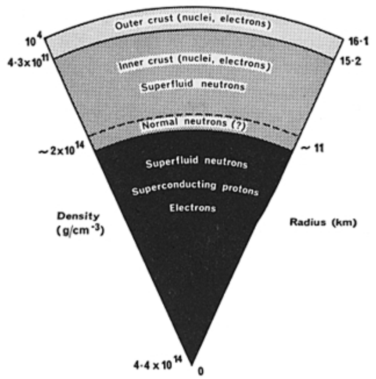
Thermal evolution

$$\frac{dS}{dt} = Q$$



The thermodynamic equilibrium in any layer but

- Different compositions
- Different equations of state



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The thermodynamic equilibrium of neutron star

- Grand canonical ensembles in layers
- Minima of thermodynamic potentials of layers
- Parametric EoS's
- Phase equilibria conditions
- Phase stratification due to the gravity

Dense hadronic stellar matter is much more difficult to handle than the dense hadronic matter from the HIC

Example

Level density for nuclei

$$N_j = \frac{1}{2\pi^2} \sum_{j=1} g_j(T) \int_0^\infty dp\, p^2 \, \exp\left\{ -\frac{E_j - \mu_j}{T} \right\}$$

with

$$g_j(T) = g_j^{(0)}(T) + \frac{c_1}{A_j^{5/3}} \int_0^\infty dE\, e^{-E/T}\, e^{2\sqrt{a_j E}}$$

where

$$a_j = \frac{A_j}{8} \left(1 - \frac{c_2}{A_j^{1/3}} \right)$$

Conclusions

- Statistical models in HIC: success story
- Statistical models in stars: still many unknowns

Challenge for theoreticians